B.Tech CSE
Sub:- IEMA
Sem:- 5th

## Assignment ⇒ 4

Q1) Represent Beta function.

Ans The Beta function is denoted by & defined by:-

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \quad \text{where } m, n > 0$$

Q2) Represent Gamma ~~function~~ formula.

Ans The Gamma formula is denoted & defined by

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx, \quad n > 0$$

Q3) Generate Riemann Integration of $\int_0^b 3x\,dx$

Slon Let, $[0, b]$ divided into n equal part of partitation

⇒ $h \Rightarrow \frac{b-0}{n} = \frac{b}{n}$

⇒ $[0, b)\ P = \left(0, \frac{b}{n}, \frac{2b}{n}, \frac{3b}{n} \cdots \frac{nb}{n}\right)$ $\frac{ib}{n}$

⇒ $3\int_0^b x\,dx \Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} 3 f(t_i).h$

⇒ $\lim_{n\to\infty} \sum_{i=1}^{n} 3\frac{ib}{n}\cdot\frac{b}{n} \Rightarrow \lim_{n\to\infty} 3\frac{b^2}{n^2}\left(1+2+3\cdots+n\right)$

$$\lim_{n\to\infty} \frac{3b^2}{n^2}\left[\frac{n(n+1)}{2}\right]$$

$$\lim_{n\to\infty} \frac{3b^2}{\cancel{n^2}}\left[\frac{\cancel{n^2}(1+1/n)}{2}\right]$$

$$\Rightarrow \quad \frac{3b^2}{2} \quad \underline{\text{Ans}}$$

Q. Generate Riemann Integration of $\int_0^a x^2\,dx$.

Sol^n interval. $[0, a]$

$$h = \frac{a-0}{n} = \frac{a}{n}$$

$$\boxed{h = \frac{a}{n}}$$

Let, partition.

$$P = \left[x_0 = 0, \frac{a}{n}, \frac{2a}{n}, \frac{3a}{n}, \ldots \frac{(r-1)a}{n}, \frac{ra}{n}, \frac{na}{n}\right]$$

General interval $\left[\frac{(r-1)a}{n}, \frac{r}{n}\right]$

$m_r$ — Infimum

$M_r$ — Supremum

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$$L(P,f) = \sum_{r=1}^{n} m_r \, \delta r_r \qquad \because f(x) = x^2$$

$$= \sum_{r=1}^{n} \frac{(r-1)^2 a^2}{n^2} \cdot \frac{a}{n}$$

$$= \frac{1}{n^3} a^3 \sum_{r=1}^{n} (r-1)^2$$

$$= \frac{a^3}{n^3} \left[ 0^2 + 1^2 + 2^3 + 3^2 \cdots (n-1)^2 \right]$$

$$= \frac{a^3}{n^3} \sum (n-1)^2$$

$$\because \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \sum (n+1)^2 = \frac{n((n-1)-1)(2(n-1)+1)}{6}$$

$$= \frac{n(n-1)[2n-1]}{6}$$

$$\Rightarrow \frac{a^3}{n^3} \cdot \frac{n[n-1][2n-1]}{6}$$

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$$\int_0^{a} x^2 dx = \lim_{n\to\infty} L(P,f)$$

$$\lim_{n\to\infty} \frac{n(n-1)(2n-1)}{6} \cdot \frac{a^3}{n^3}$$

$$\lim_{n\to\infty} \frac{\cancel{n^3}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{\cancel{n^3}} \cdot a^3$$

$$\lim_{n\to\infty} \frac{\cancel{2}1}{\cancel{6}\,3} \cdot a^3 \Rightarrow \frac{a^3}{3}$$

$$\Rightarrow \int_0^{a} x^2 dx = \frac{a^3}{3}$$

Q5) Evaluate $\iint x+y \, dx\, dy$, where the region of integration is $3x+2y \le 1$ in the positive quadrant.

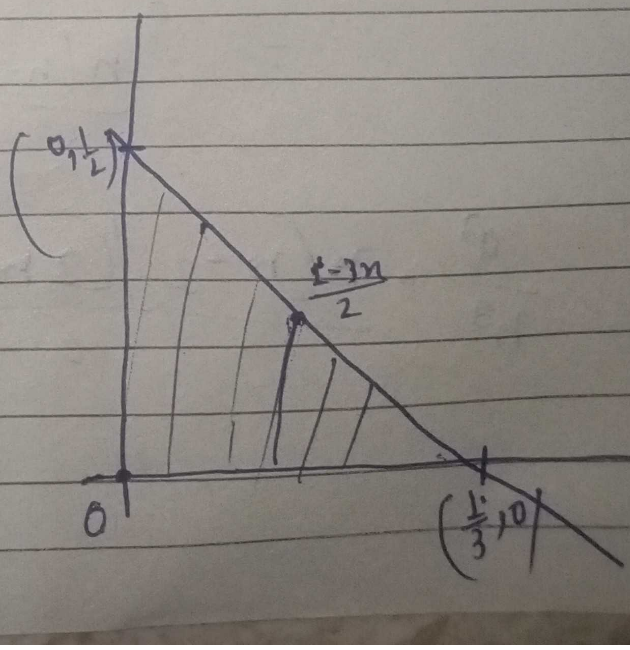
Sol<sup></sup>n Given that,

$$3x+2y \le 1$$

$$3x+2y = 1$$

$$y = \frac{1-3x}{2}$$

| $x$ | $0$ | $1/3$ | |
|---|---|---|---|
| $y$ | $1/2$ | $0$ | |



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$$\Rightarrow \int_{x=0}^{1/3}\int_{y=0}^{\frac{1-3x}{2}} (x+y)\, dx\, dy.$$

$$\Rightarrow \int_{x=0}^{1/3}\left(\int_{y=0}^{\frac{1-3x}{2}} (x+y)\, dy\right) dx$$

$$\Rightarrow \int_{x=0}^{1/3}\left(xy+\frac{y^2}{2}\right)_0^{\frac{1-3x}{2}} dx \Rightarrow \int_{x=0}^{1/3} x\left(\frac{1-3x}{2}\right)+\frac{\left(\frac{1-3x}{2}\right)^2}{2} dx$$

$$\Rightarrow \int_{x=0}^{1/3} \frac{x-3x^2}{2}+\frac{\frac{(1^2+9x^2-6x)}{8}}{2} dx$$

$$\Rightarrow \int_{x=0}^{1/3}\left(\frac{4x-12x^2+1+9x^2-6x}{8}\right) dx$$

$$\Rightarrow \int_{x=0}^{1/3} \frac{-3x^2-2x+1}{8} dx$$

$$\Rightarrow \frac{1}{8}\left[4\frac{x^2}{2}-12\frac{x^3}{3}+x+9\frac{x^3}{3}-6\frac{x^2}{2}\right]_0^{1/3}$$

$$\Rightarrow \frac{1}{8}\left[2x^2-4x^3+x+3x^3-3x^2\right]_0^{1/3}$$

$$\Rightarrow \frac{1}{8}\left[2\left(\frac{1}{3}\right)^2-4\left(\frac{1}{3}\right)^3+\frac{1}{3}+3\left(\frac{1}{3}\right)^3-3\left(\frac{1}{3}\right)^2\right]$$

$$\Rightarrow \frac{1}{8}\left[\frac{2}{9}-\frac{4}{27}+\frac{1}{3}+\frac{\cancel{3}^{1}}{\cancel{27}_{9}}-\frac{3}{9}\right]$$

$$\Rightarrow \frac{1}{8}\left[\frac{2}{9}-\frac{4}{27}+\frac{1}{3}+\frac{1}{9}-\frac{1}{3}\right]$$

$$\Rightarrow \frac{1}{8}\left[\frac{6-4+9+3-9}{27}\right]$$

$$\Rightarrow \frac{1}{8}\times\frac{5}{27} \Rightarrow \frac{5}{216} \text{ Ans}$$